Note that the transformation in $f\left(\frac{1}{5}x\right)$ is applied to the *argument* of function f, *inside* the parentheses. The transformation in $3f(x)$ is applied *outside* the parentheses, to the *value* of the function. For this reason the transformations are given the names **inside transformation** and **outside transformation,** respectively. An inside transformation affects the graph in the horizontal direction, and an outside transformation affects the graph in the vertical direction.

You may ask, "Why do you *multiply* by the y-dilation and *divide* by the x-dilation?" You can see the reason by substituting y for $g(x)$ and dividing both sides of the equation by 3:

$$y = 3f\left(\frac{1}{5}x\right)$$

$$\frac{1}{3}y = f\left(\frac{1}{5}x\right) \qquad \text{Divide both sides by 3 } \left(\text{or multiply by } \tfrac{1}{3}\right).$$

You actually divide by *both* dilation factors, y by the y-dilation and x by the x-dilation.

Translations

The translations in Figure 1-3a that transform $f(x)$ to $h(x)$ are shown again in Figure 1-3e. To figure out what translation has been done, ask yourself, "To where did the point at the origin move?" As you can see, the center of the semicircle, initially at the origin, has moved to the point (4, 2). So there is a horizontal translation of 4 units and a vertical translation of 2 units.

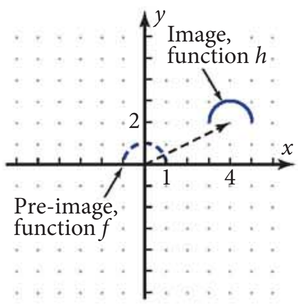

Figure 1-3e

To get a vertical translation of 2 units, add 2 to each y-value:

$$y = 2 + f(x)$$

To get a horizontal translation of 4 units, note that what was happening at $x = 0$ in function f has to be happening at $x = 4$ in function h. Again, substituting v for the argument of f gives

$$h(x) = 2 + f(v)$$

$$x = v + 4$$

$$x - 4 = v$$

$$h(x) = 2 + f(x - 4) \qquad \text{Substitute } x - 4 \text{ as the argument of } f.$$